N-TARTIBLI DETERMINANTLARNI HISOBLASHDA BA'ZI BIR XOSSALARDAN FOYDALANISH

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Annotatsiya. Maqolada yuqori tartibli determinantlar va ularni hisoblash usullari haqida so'z borgan. Ikkinchi va uchinchi tartibli determinantlarni hisoblashning bir nechta qulay usullari mavjud bo'lib, to'rtinchi va undan yuqori tartibli determinantlarni hisoblashda esa algebraik to'ldiruvchi va minorlar usulidan foydalaniladi. Bu usul bosqichma-bosqich tartib kamaytirish orqali bajariladi toki bizga ma'lum bo'lgan ikkinchi yoki uchinchi tartibga kelmaguncha. Tartib yuqorilagani sari determinantni hisoblash qiyinlashib boraveradi. Ushbu maqolada yuqori tartibli determinantlarni hisoblashda qulaylik yaratadigan xossalar va ularning isbotlari o'quvchi-talabalarga ma'lum bo'lgan uchinchi tartibli determinantlar misolida keltirilgan.

Kalit so'zlar: determinant, matritsa, n-tartibli determinant, "uchburchaklar usuli", "Sarryus usuli", xossa, satr, ustun, element, transponirlash, qiymat.

USING SOME PROPERTIES IN CALCULATING N-ORDER DETERMINANTS

Abstract. The article discusses high-order determinants and methods for calculating them. While there are several convenient methods for calculating second- and third-order determinants, the algebraic complement and minor methods are used for fourth-order and higher determinants. These methods involve a step-by-step reduction of the order until reaching the second or third order, which are commonly known. As the order increases, the complexity of calculating the determinant also increases. This article presents properties that facilitate the calculation of high-order determinants and their proofs using examples of third-order determinants familiar to students.

Keywords: determinant, matrix, n-order determinant, "triangle method," "Sarrus method," property, row, column, element, transposition, value.

ИСПОЛЬЗОВАНИЕ НЕКОТОРЫХ СВОЙСТВ ПРИ ВЫЧИСЛЕНИИ ОПРЕДЕЛИТЕЛЕЙ N-ГО ПОРЯДКА

Аннотация. В статье рассматриваются определители высокого порядка и методы их вычисления. Существуют несколько удобных методов для вычисления определителей второго и третьего порядка, однако для определителей четвертого и более высокого порядка используются методы алгебраического дополнения и миноров. Эти методы предполагают пошаговое снижение порядка до второго или третьего, которые являются широко известными. По мере увеличения порядка сложность вычисления определителя также возрастает. В данной статье представлены свойства, упрощающие вычисление определителей высокого порядка, и их доказательства на примерах определителей третьего порядка, известных студентам.

Ключевые слова: определитель, матрица, определитель n-го порядка, «метод треугольников», «метод Саррюса», свойство, строка, столбец, элемент, транспонирование, значение.

Determinant tushunchasidan dastlab chiziqli tenglamalar sistemasini yechishda foydalanilgan bo‘lib, keyinchalik ular matematikaning bir qancha masalalarini yechishga, jumladan xos sonlarni topishga, differensial tenglamalarni yechishga, vektor hisobiga keng tatbiq etildi.

Determinant faqat kvadrat matritsalar uchun o‘rinli bo‘lib, uni hisoblashning bir qancha usullari mavjud. Ikkinchi va uchinchi tartibli matritsalarni “Uchburchaklar usuli” va “Sarryus qoidalari” orqali hisoblash qulaydir.

$$detA_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{21} \cdot a_{12}$$

$$detA_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{31} \cdot a_{12} \cdot a_{23} + a_{13} \cdot a_{21} \cdot a_{32} -$$

$$-a_{31} \cdot a_{22} \cdot a_{13} - a_{11} \cdot a_{23} \cdot a_{32} - a_{33} \cdot a_{12} \cdot a_{21}$$

To‘rtinchi va undan yuqori tartibli determinantlarni hisoblashda esa minorlar va algebraik to‘ldiruvchilardan foydalanamiz.

$$detA_n = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} = a_{11} \cdot A_{11} + a_{12} \cdot A_{12} + \cdots + a_{1n} \cdot A_{1n} =$$

$$= a_{11} \cdot (-1)^{1+1} \cdot \begin{bmatrix} a_{22} & \cdots & a_{2n} \\ \vdots & \ddots & \vdots \\ a_{n2} & \cdots & a_{nn} \end{bmatrix} + a_{12} \cdot (-1)^{12} \cdot \begin{bmatrix} a_{21} & \cdots & a_{2n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} + \cdots +$$

$$+a_{1n} \cdot (-1)^{1+n} \cdot \begin{bmatrix} a_{21} & \cdots & a_{2(n-1)} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{n(n-1)} \end{bmatrix} = \cdots$$

Hisob shu tarzda davom ettiriladi toki minorlar tartibi ikkinchi yoki uchinchi tartibga kelmaguncha. Quyida uchinchi tartibli determinantning minorlari va algebraik to‘ldiruvchilarini topamiz:

$$detA_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Minorlarni M_{ij} orqali algebraik to‘ldiruvchilarni esa A_{ij} orqali belgilaymiz.

$$A_{ij} = (-1)^{i+j} \cdot M_{ij}$$

Birinchi satr elementlarining minorlari va algebraik to‘ldiruvchilarini topaylik.

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}, M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = (-1)^2 \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = (-1)^3 \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = (-1)^4 \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$detA_3 = a_{11} \cdot A_{11} + a_{12} \cdot A_{12} + a_{13} \cdot A_{13}$$

$$detA_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} +$$

$$+a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{11} \cdot (a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) - a_{12} \cdot (a_{21} \cdot a_{33} - a_{23} \cdot a_{31}) +$$

$$+a_{13} \cdot (a_{21} \cdot a_{32} - a_{22} \cdot a_{31}) = a_{11} \cdot a_{22} \cdot a_{33} - a_{11} \cdot a_{23} \cdot a_{32} - a_{12} \cdot a_{21} \cdot a_{33} +$$

$$+a_{12} \cdot a_{23} \cdot a_{31} + a_{13} \cdot a_{21} \cdot a_{32} - a_{13} \cdot a_{22} \cdot a_{31} = a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{13} \cdot a_{21} \cdot a_{32} - a_{11} \cdot a_{23} \cdot a_{32} - a_{12} \cdot a_{21} \cdot a_{33} - a_{13} \cdot a_{22} \cdot a_{31}$$

Har ikki usulda ham determinant bir xil ko‘rinishda yoyildi. Qolgan elementlarning ham minorlari va algebraik to‘ldiruvchilari ham huddi shu tartibda topiladi va ular uchun ham quyidagi formulalar o‘rinli hisoblanadi.

$$detA_3 = a_{21} \cdot A_{21} + a_{22} \cdot A_{22} + a_{23} \cdot A_{23}$$

$$detA_3 = a_{31} \cdot A_{31} + a_{32} \cdot A_{32} + a_{33} \cdot A_{33}$$

$$detA_3 = a_{11} \cdot A_{11} + a_{21} \cdot A_{21} + a_{31} \cdot A_{31}$$

$$detA_3 = a_{12} \cdot A_{12} + a_{22} \cdot A_{22} + a_{32} \cdot A_{32}$$

$$detA_3 = a_{13} \cdot A_{13} + a_{23} \cdot A_{23} + a_{33} \cdot A_{33}$$

Tartib ortib borgan sari determinantni hisoblash ham tobora qiyinlashib boraveradi.

Quyida n-tartibli determinantni hisoblashda qulaylik tug‘diruvchi ba’zi bir xossalardan foydalanamiz.

1-xossa: Transponirlash natijasida determinantning qiymati o‘zgarmaydi. Ya’ni satr va ustun elementlarini o‘rnini almashtirib yozishdan hosil bo‘lgan determinantning qiymati o‘zgarmaydi.

$$detA_n = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{n1} \\ \vdots & \ddots & \vdots \\ a_{1n} & \cdots & a_{nn} \end{bmatrix} = detA^T$$

Isbot: Bu xossalarning isbotini uchinchi tartibli determinantlarda ko‘raylik.

$$detA_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{31} \cdot a_{12} \cdot a_{23} + a_{13} \cdot a_{21} \cdot a_{32} -$$

$$-a_{31} \cdot a_{22} \cdot a_{13} - a_{11} \cdot a_{23} \cdot a_{32} - a_{33} \cdot a_{12} \cdot a_{21}$$

$$detA_3{}^T = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{31} \cdot a_{12} \cdot a_{23} + a_{13} \cdot a_{21} \cdot a_{32} -$$

$$-a_{31} \cdot a_{22} \cdot a_{13} - a_{11} \cdot a_{23} \cdot a_{32} - a_{33} \cdot a_{12} \cdot a_{21}$$

Demak, $detA_3 = detA_3{}^T$.

2-xossa: Determinantning istalgan ikkita satr yoki ikkita ustun elementlari o‘rinlari almashtirilganda, determinantning qiymati qarama-qarshisiga o‘zgaradi.

$$detA_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$detA_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{31} \cdot a_{12} \cdot a_{23} + a_{13} \cdot a_{21} \cdot a_{32} -$$

$$-a_{31} \cdot a_{22} \cdot a_{13} - a_{11} \cdot a_{23} \cdot a_{32} - a_{33} \cdot a_{12} \cdot a_{21}$$

$$\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{pmatrix} a_{11} \cdot a_{22} \cdot a_{33} + a_{31} \cdot a_{12} \cdot a_{23} + a_{13} \cdot a_{21} \cdot a_{32} - \\ a_{31} \cdot a_{22} \cdot a_{13} - a_{11} \cdot a_{23} \cdot a_{32} - a_{33} \cdot a_{12} \cdot a_{21} \end{pmatrix}$$

3-xossa: Agar determinantning ikkita satri bir xil yoki ikkita ustuni bir xil elementlardan tuzilgan bo‘lsa, uning qiymati nolga teng bo‘ladi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot a_{12} \cdot a_{33} + a_{31} \cdot a_{12} \cdot a_{13} + a_{13} \cdot a_{11} \cdot a_{32} -$$

$$-a_{31} \cdot a_{12} \cdot a_{13} - a_{11} \cdot a_{32} \cdot a_{13} - a_{33} \cdot a_{11} \cdot a_{12} = 0$$

4-xossa: Agar determinantning biror satri yoki biror ustuni μ songa ko'paytirilsa, determinant ham shu songa ko'paytiriladi va aksincha.

$$\begin{vmatrix} \mu \cdot a_{11} & \mu \cdot a_{12} & \mu \cdot a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \mu \cdot detA_3$$

$$\begin{vmatrix} \mu \cdot a_{11} & \mu \cdot a_{12} & \mu \cdot a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \mu \cdot a_{11} \cdot a_{22} \cdot a_{33} + a_{31} \cdot \mu \cdot a_{12} \cdot a_{23} +$$

$$+\mu \cdot a_{13} \cdot a_{21} \cdot a_{32} - a_{31} \cdot a_{22} \cdot \mu \cdot a_{13} - \mu \cdot a_{11} \cdot a_{23} \cdot a_{32} - a_{33} \cdot \mu \cdot a_{12} \cdot a_{21}$$

$$= \mu \cdot (a_{11} \cdot a_{22} \cdot a_{33} + a_{31} \cdot a_{12} \cdot a_{23} + a_{13} \cdot a_{21} \cdot a_{32} - a_{31} \cdot a_{22} \cdot a_{13} - a_{11} \cdot a_{23} \cdot a_{32} - a_{33} \cdot a_{12} \cdot a_{21}) = \mu \cdot detA_3$$

5-xossa: Agar determinantning ikkita satri yoki ikkita ustuni o'zaro proporsional bo'lsa, determinant qiymati nolga tengdir.

$$\begin{vmatrix} \mu \cdot a_{11} & \mu \cdot a_{12} & \mu \cdot a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

Isbot: Bu xossaning isbotini 3- va 4-xossalarni qo'llagan holda bajariladi. Ya'ni 4-xossaga ko'ra μ son determinantdan tashqariga chiqariladi. 3-xossaga ko'ra determinantning birinchi va ikkinchi satr elementlari bir xil ekanligidan uning qiymati nolga tengligi isbotlanadi.

6-xossa: Agar determinantning biror satri yoki biror ustuni elementlari noldan iborat bo'lsa, determinant nolga teng bo'ladi.

Isbot: $\begin{vmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{vmatrix} = a_{11} \cdot a_{22} \cdot 0 + a_{31} \cdot a_{12} \cdot 0 + 0 \cdot a_{21} \cdot a_{32} - a_{31} \cdot a_{22} \cdot 0 - a_{11} \cdot a_{32} \cdot 0 - 0 \cdot a_{12} \cdot a_{21} = 0$

Xossalarning isbotini 3-tartibli determinantlar misolida ko'rdik. Bu xossalar n-tartibli determinantlar uchun ham o'rinli.

Yuqori tartibli determinantlarni hisoblash har doim ham qulay amalga oshavermaydi. Ba'zi determinantlar qiymatini esa oldindan aytish mushkul hisoblanadi. Bunday determinantlarni hisoblashda ushbu maqolada keltirilgan xossalardan foydalanish o'quvchi-talabalarga qulaylik tug'diradi, determinant qiymatini bevosita hisob-kitobdan oldin qanday natijaga erisha olishini oldindan aytish mumkin hisoblanadi va ularni matematikaga bo'lgan qiziqishlarini yanada orttiradi.

Foydalanilgan adabiyotlar ro'yhati

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