IKKINCHI TUR BUZILADIGAN GIPERBOLIK TIPDAGI TENGLAMA n=1 BO'LGAN XOL UCHUN KOSHI TIPIDAGI MASALA

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Annotatsiya Ushbu maqola ikkinchi tur buzilish chizig'iga ega bo'lgan giperbolik tenglamalar uchun masalalarni bayon qilishga bag'ishlangan bo'lib, buziladigan ikkinchi tur giperbolik tenglama uchun kichik had koeffisienti $-1/2$ ga teng bo'lganda Koshi tipidagi masala qo'yilishi va bu masala uchun yechimni topishdan iborat.

Ushbu

$$L_{-1/2}(U)\equiv U_{xx}+yU_{yy}-\frac{1}{2}U_y=0 \qquad (1)$$

tenglamani $OB: x-2\sqrt{-y}=0$, $AB: x+2\sqrt{-y}=1$ va $OA: y=0$ xarakteristikalar bilan chegaralangan D sohada qaraylik.

Ta'kidlash joizki, Krikunov Yu.M. [5], hamda Xayrullin R.S. [6,7] tomonidan giperbolik qismi (1) tenglamadan iborat bo'lgan aralash elliptiko-giperbolik tenglama uchun turli chegaraviy masalalar o'rganilgan. Giperbolik qismi (1) tenglamadan iborat bo'lgan aralash parabolo-giperbolik tenglama uchun Trikomi masalasi [6] ishda o'rganilgan.

Masala (Koshi tipidagi masala). *D sohada (1) tenglamani va ushbu*

$$u(x,0)=\tau(x),\, x\in[0,1];\;\; \lim_{y\to-0}\sqrt{-y}\left(\partial^2/\partial y^2\right)u=\nu(x),\;\; x\in(0,1) \qquad (2)$$

boshlang'ich shartlarni qanoatlantiruvchi $u(x,y)\in C(\overline{D})\cap C^2(D)$ funksiya topilsin, bu yerda $\tau(x)$ va $\nu(x)$ - berilgan uzluksiz funksiyalar.

Masalaning yechilishi. Ma'lumki (1) tenglamaning umumiy yechimi [Krikunov]

$$u=2\left[\varphi\left(x-2\sqrt{-y}\right)+\varphi\left(x+2\sqrt{-y}\right)\right]+4(-y)^{\frac{1}{2}}\left[\varphi'\left(x-2\sqrt{-y}\right)-\varphi'\left(x+2\sqrt{-y}\right)\right]+$$

$$+(-y)^{3/2}\int_0^1\psi\left[x-2\sqrt{-y}\,(1-2z)\right]\left[z(1-z)\right]dz \qquad (3)$$

ko'rinishda bo'ladi, bu yerda $\varphi(x)$, $\psi(x)$ funksiyalar ihtiyoriy funksiyalar. (3)ni (2) shartning birinchisiga qo'yib

$$\varphi(x)=\frac{1}{4}\tau(x)$$

tenglikka ega bo'lamiz. Buni (3)ga qo'yib

$$u=\frac{1}{2}\left[\tau\left(x-2\sqrt{-y}\right)+\tau\left(x+2\sqrt{-y}\right)\right]+(-y)^{\frac{1}{2}}\left[\tau'\left(x-2\sqrt{-y}\right)-\tau'\left(x+2\sqrt{-y}\right)\right]+$$

$$+(-y)^{3/2}\int_0^1 \psi\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz \qquad (4)$$

ga ega bo'lamiz. Endi, (4) funksiyani (2) shartning ikkinchisiga qo'yish uchun (4) funksiyadan y bo'yicha ikki marta hosila olamiz

$$u_{yy}=\frac{4}{\sqrt{-y}}\left[\tau'''\left(x-2\sqrt{-y}\right)-\tau'''\left(x+2\sqrt{-y}\right)\right]+$$

$$+\frac{3}{4\sqrt{-y}}\int_0^1 \psi\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz+$$

$$-\frac{3}{2}\int_0^1 \psi'\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz+\int_0^1 \psi'\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz+$$

$$+\sqrt{-y}\int_0^1 \psi''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz. \qquad (5)$$

Olingan hosilani $\sqrt{-y}$ ga ko'paytirib, so'ngra $y\to -0$ da limitga o'tib

$$\lim_{y\to -0}\sqrt{-y}\left(\partial^2/\partial y^2\right)u=4\lim_{y\to -0}\left[\tau'''\left(x-2\sqrt{-y}\right)-\tau'''\left(x+2\sqrt{-y}\right)\right]+$$

$$+\frac{3}{4}\lim_{y\to -0}\int_0^1 \psi\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz+$$

$$-\frac{3}{2}\lim_{y\to -0}\sqrt{-y}\int_0^1 \psi'\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz+$$

$$+\lim_{y\to -0}\sqrt{-y}\int_0^1 \psi'\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz-$$

$$-\lim_{y\to -0}y\int_0^1 \psi''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz=$$

$$4\left[\tau'''(x)-\tau'''(x)\right]+\frac{3}{4}\psi(x)\int_0^1 z(1-z)\,dz=\frac{1}{8}\psi(x) \qquad (6)$$

ga ega bo'lamiz. (6) va (2) shartning ikkinchisidan

$$\psi(x)=8\nu(x),\ \ x\in(0,1)$$

tenglikni hosil qilamiz. Olingan tenglikni (4) funksiyaga qo'yib {(1), (2)} masala yechimini

$$u=\frac{1}{2}\left[\tau\left(x-2\sqrt{-y}\right)+\tau\left(x+2\sqrt{-y}\right)\right]+(-y)^{\frac{1}{2}}\left[\tau'\left(x-2\sqrt{-y}\right)-\tau'\left(x+2\sqrt{-y}\right)\right]+$$

$$+8(-y)^{3/2}\int_0^1 \nu\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz \qquad (7)$$

hosil qilamiz.

Teorema 1. *Agar* $\tau(x)\in C^3[0,1]$, $\nu(x)\in C^2[0,1]$ *bo'lsa {(3.1.1), (3.1.2)} masala (3.1.7) yagona yechimga ega bo'ladi.*

Isbot. Avvalo (7) funksiya (1) tenglamaning yechimi ekanini tekshiramiz. Buning uchun (7) funksiyadan kerakli hosilalarni olamiz

$$u_{xx}=\frac{1}{2}\left[\tau''\left(x-2\sqrt{-y}\right)+\tau''\left(x+2\sqrt{-y}\right)\right]+$$

$$+(-y)^{\frac{1}{2}}\left[\tau'''\left(x-2\sqrt{-y}\right)-\tau'''\left(x+2\sqrt{-y}\right)\right]+$$

$$+8(-y)^{3/2}\int_0^1\nu''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz,$$

$$u_y=\left[\tau''\left(x-2\sqrt{-y}\right)+\tau''\left(x+2\sqrt{-y}\right)\right]-$$

$$-12\sqrt{-y}\int_0^1\nu\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz+$$

$$+8y\int_0^1\nu'\left[x-2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz \qquad (8)$$

$$u_{yy}=\frac{1}{\sqrt{-y}}\left[\tau'''\left(x-2\sqrt{-y}\right)-\tau'''\left(x+2\sqrt{-y}\right)\right]+$$

$$+\frac{3}{4\sqrt{(-y)}}\int_0^1\nu\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz-$$

$$-\frac{3}{2}\int_0^1\nu'\left[x-2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz-$$

$$-\int_0^1\nu'\left[x-2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz-$$

$$-\sqrt{-y}\int_0^1(1-2z)^2\,\nu''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz+$$

$$+8(-y)^{3/2}\int_0^1\nu''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz \qquad (9)$$

(7), (8), (9)larni (1) ga qo'yish natijasida quyidagi tenglikga kelamiz

$$L_{-1/2}(U)=\sum_{k=1}^{11}A_k,$$

bu yerda

$$A_1=\frac{1}{2}\left[\tau''\left(x-2\sqrt{-y}\right)+\tau''\left(x+2\sqrt{-y}\right)\right],$$

$$A_2=(-y)^{\frac{1}{2}}\left[\tau'''\left(x-2\sqrt{-y}\right)-\tau'''\left(x+2\sqrt{-y}\right)\right],$$

$$A_3 = 8(-y)^{3/2}\int_0^1 \nu''\left[x - 2\sqrt{-y}(1-2z)\right](1-2z)^2\left[z(1-z)\right]dz,$$

$$A_4 = -\sqrt{-y}\left[\tau'''\left(x - 2\sqrt{-y}\right) - \tau'''\left(x + 2\sqrt{-y}\right)\right],$$

$$A_5 = -6\sqrt{-y}\int_0^1 \nu\left[x - 2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz,$$

$$A_6 = -20y\int_0^1 \nu'\left[x - 2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz,$$

$$A_7 = -8(-y)^{\frac{3}{2}}\int_0^1 (1-2z)^2\nu''\left[x - 2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz,$$

$$A_8 = -\frac{1}{2}\left[\tau''\left(x - 2\sqrt{-y}\right) + \tau''\left(x + 2\sqrt{-y}\right)\right],$$

$$A_9 = 6\sqrt{-y}\int_0^1 \nu\left[x - 2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz,$$

$$A_{10} = 4y\int_0^1 \nu'\left[x - 2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz,$$

Bu yerda

$$A_1 + A_8 = 0, \quad A_2 + A_4 = 0, \; A_5 + A_9 = 0,$$

$$A_3 + A_6 + A_{10} + A_7 = 8(-y)^{3/2}\int_0^1 \nu''\left[x - 2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz -$$

$$-20y\int_0^1 \nu'\left[x - 2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz +$$

$$+4y\int_0^1 \nu'\left[x - 2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz -$$

$$-8(-y)^{\frac{3}{2}}\int_0^1 (1-2z)^2\nu''\left[x - 2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz =$$

$$= 8(-y)^{3/2}\int_0^1 \nu''\left[x - 2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz -$$

$$-16y\int_0^1 \nu'\left[x - 2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz -$$

$$-8(-y)^{\frac{3}{2}}\int_0^1 (1-2z)^2\nu''\left[x - 2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz =$$

$$=8(-y)^{3/2}\int_0^1 v''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz-$$

$$-16y\int_0^1 v'\left[x-2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz-$$

$$-8(-y)^{3/2}\int_0^1 v''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz+$$

$$+32(-y)^{\frac{3}{2}}\int_0^1 (1-2z)v''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz=$$

$$=-16y\int_0^1 v'\left[x-2\sqrt{-y}(1-2z)\right](1-2z)\left[z(1-z)\right]dz+$$

$$+32(-y)^{\frac{3}{2}}\int_0^1 v''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]^2 dz=$$

$$=-16y\frac{1}{2}\Big\{\left[z(1-z)\right]^2 v'\left[x-2\sqrt{-y}(1-2z)\right]+$$

$$+4\sqrt{-y}\int_0^1 v''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]^2 dz\Big\}+$$

$$+32(-y)^{\frac{3}{2}}\int_0^1 (1-2z)v''\left[x-2\sqrt{-y}(1-2z)\right]\left[z(1-z)\right]dz=0$$

bo’lganidan $L_{-1/2}(U)=0$ ekani kelib chiqadi

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